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A Novel Trajectory Tracking Control for a Ball and Plate System

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Abstract

A new control strategy is proposed for the trajectory tracking problem of the ball and plate system. The control protocol is designed by transforming the system model into a fully-actuated system, and the sampling time of the discrete system is divided into multiple modules in turn by controllable coefficient. The stability of the system is analyzed by mathematical analysis method, and finally the correctness of the proposed protocol is verified by simulation experiments.

Keywords

Ball and plate system, fully-actuated, trajectory tracking.

1. Introduction

The ball and plate system is a multi-variable, nonlinear and strongly coupled controlled system. As a mechanical structure system with two degrees of freedom, it is widely used in the research of dynamic system and control process of modern control theory. The system has been practically applied in the research of some dynamic systems, such as robots, rocket systems, unmanned aerial vehicles. Most laboratory-based ball and plate control systems have inherent nonlinearity and instability due to the irregular oscillation of the platform and the positioning of the ball[1]. Therefore, the trajectory tracking problem of ball and plate system is a typical control challenge. The focus of this paper is to design a control mechanism that can ensure the system has good stability and trajectory tracking performance.

Some researches have used classical techniques to deal with the trajectory tracking problem of the systems. Galvan-colmenares et al. proposed an asymptotically stable normal proportional derivative (PD) control method considering nonlinear compensation[2]. Oravec et al. adopted a model predictive control method to implement the trajectory tracking control problem of ball and plate system[3]. Umar et al. used the Hinfinity control method[4]. Florian Köpf et al. designed model-free data-based adaptive dynamic programming tracking controller for a large-scale ball and plate system[5]. There has been a lot of research on the ball and plate system[6-7], and this paper designs a discrete and modularized control method. Duan Guangren introduced the concept of fully-actuated system and its advantages in controller design[8]. The system is transformed into an fully-actuated system in an iterative manner, and the maximum controllable coefficient of the all-drive system is taken as the module interval. The stability of the protocol is analyzed by mathematical analysis, and the effectiveness of the method is finally verified by simulation.

2. Control system modeling analysis

Since the X-axes and Y-axes of the ball and plate system are perpendicular to each other, it can be regarded as two ball and beam system in perpendicular directions, that is, the two-dimensional extension of the ball and beam system. To simplify the model, assume that X-Y-Z is the world coordinate system connected to the pedestal, and X''-Y''-Z'' is the local coordinate system fixed on the ball and beam system. The inclination of the ball disc relative to the X-axis is q_x and the inclination of the Y-axis is q_y, and the counterclockwise direction is positive. The movement of the ball on the plate is decomposed into the X-axis and Y-axis directions, as shown in Figure 1.

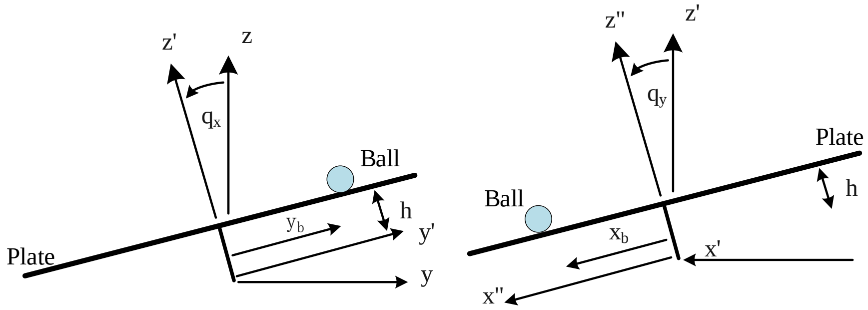

Figure 1: Two-dimensional decomposition of ball and plate system

Due to the complex nonlinear characteristics of the ball and plate system, it is difficult to obtain an accurate mathematical model. The following assumptions are made without affecting the system characteristics during the modeling process:

(1) In any case the ball is in contact with the plane;

(2) Area and inclination limitations of the plate are not considered;

(3) The ball does not slide and rotate around the vertical center axis on the plate;

(4) The origin of the plate coordinate system and the world coordinate system coincide;

(5) All frictions are ignored.

Some ball and plate system parameters are shown in Table 1.

Table 1: Ball and plate system parameters table

Parameters	meaning	unit
m	Weight of the ball	kg
r_b	Radius of ball	m
x, y	The displacement of the ball in the x and y directions of the plate coordinate system	m
θ_x, θ_y	The inclination of the plate in the x and y directions	rad
r	The distance of the ball from the origin in the plate coordinate system	m
r_p	The distance of the ball from the origin in world coordinates	m
ω_x, ω_y	The rotation speed of the ball in the x and y directions of the plate coordinates	rad/s
J_b, J_p	Moment of inertia of ball and plate	$kg \cdot m^2$

As shown in Figure 2, when the inclination of plate is small, the following relationship exists:

$$q_x = \frac{d_x}{L_x}\theta_x,\ q_y = \frac{d_y}{L_y}\theta_y \tag{2-1}$$

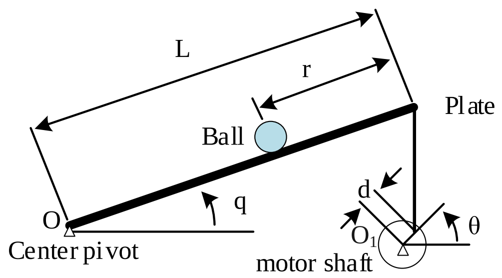

Figure 2: the structure of the ball and plate system

The ball and plate system is modeled using the Lagrange equation. The general form of the Lagrange equation is (2-2)

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i,\quad i = 1, 2, 3, \cdots, n \tag{2-2}$$

where q_i is the generalized coordinate of the system, Q_i is the external force of the system along the direction of the generalized coordinate q_i, L is the Lagrangian of the system, and is the difference between the kinetic energy T and the potential energy V, that is, $L = T - V$.

The kinetic energy of the ball includes its own rotational kinetic energy and translational kinetic energy on the plate:

$$T_b = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}J_b(\omega_x^2 + \omega_y^2) \tag{2-3}$$

The ball does not slide during movement, $\dot{x} = r_b\omega_x$,$\dot{y} = r_b\omega_y$, then

$$T_b = \frac{1}{2}\left(m + \frac{J_b}{r_b^2}\right)(\dot{x}^2 + \dot{y}^2) \tag{2-4}$$

The kinetic energy of the plate includes its own rotational kinetic energy and the rotational kinetic energy of the ball around the support point of the plate:

$$\begin{aligned} T_p &= \frac{1}{2}J_p\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) + \frac{1}{2}(J_b + mr_p^2)\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) \\ &= \frac{1}{2}(J_p + J_b)\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) + \frac{1}{2}mr^2\left(\frac{\left(x\dot{\theta}_x + y\dot{\theta}_y\right)^2}{r^2\dot{\theta}_x^2 + r^2\dot{\theta}_y^2}\right)\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) \\ &= \frac{1}{2}(J_p + J_b)\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) + \frac{1}{2}m\left(x\dot{\theta}_x + y\dot{\theta}_y\right)^2 \end{aligned} \tag{2-5}$$

The potential energy of the plate is defined as zero, and the potential energy of the ball can be expressed as

$$V_b = mg(x\sin\theta_x + y\sin\theta_y) \tag{2-6}$$

L can be further represented by (2-4), (2-5) and (2-6)

$$
\begin{aligned}
L &= T_b + T_p - V_b \\
&= \frac{1}{2}\left(m + \frac{J_b}{r_b^2}\right)\left(\dot{x}^2 + \dot{y}^2\right) + \frac{1}{2}J_p\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) \\
&+ \frac{1}{2}\left(J_b + mr_p^2\right)\left(\dot{\theta}_x^2 + \dot{\theta}_y^2\right) - mg\left(x\sin\theta_x + y\sin\theta_y\right)
\end{aligned} \tag{2-7}
$$

Take the following variables for the Lagrange equation:

$$q_1 = x,\ q_2 = y,\ Q_i = 0 \tag{2-8}$$

Through the above derivation, a system of nonlinear differential equations can be obtained, which mathematically describes the dynamics of the system:

$$
\begin{cases}
\left(m + \frac{J_b}{r_b^2}\right)\ddot{x} - m\left(x\dot{\theta}_x^2 + y\dot{\theta}_x\dot{\theta}_y\right) + mg\sin\theta_x = 0 \\
\left(m + \frac{J_b}{r_b^2}\right)\ddot{y} - m\left(y\dot{\theta}_y^2 + x\dot{\theta}_x\dot{\theta}_y\right) + mg\sin\theta_y = 0
\end{cases} \tag{2-9}
$$

Since the inclination range of the plate is not large, which

$$\sin\theta_x \approx \theta_x, \quad \sin\theta_y \approx \theta_y \tag{2-10}$$

The ball and plate system is rolling slowly and has no constraint. The value of the coupling term $x\dot{\theta}_x^2 + y\dot{\theta}_x\dot{\theta}_y$ and $y\dot{\theta}_y^2 + x\dot{\theta}_x\dot{\theta}_y$ are very small, so (2-9) can be approximately linearized.

$$
\begin{cases}
\left(m + \frac{J_b}{r_b^2}\right)\ddot{x} + mg\theta_x = 0 \\
\left(m + \frac{J_b}{r_b^2}\right)\ddot{y} + mg\theta_y = 0
\end{cases} \tag{2-11}
$$

Define $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \theta_x$, $x_4 = \dot{\theta}_x$, $x_5 = y$, $x_6 = \dot{y}$, $x_7 = \theta_y$, $x_8 = \dot{\theta}_y$, and u_x and u_y are the inclination angle of the plate in the x direction and in the y direction, respectively. Through model decoupling, the system (2-11) can be decoupled into two subsystems along the x and y directions, and the simplified system state equations can be obtained.

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{mg}{m + \frac{J_b}{r_b^2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_x \tag{2-12}
$$

$$
\begin{bmatrix} \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{mg}{m + \frac{J_b}{r_b^2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_y \tag{2-13}
$$

Connect the control input to a zero-order holder and hold the control input constant for the sampling period T. (2-12) and (2-13) further can be obtained

$$\begin{matrix} X_i(k+1)=GX_i(k)+Hu_i(k) \\ Y_i(k)=CX_i(k) \end{matrix}, \text{for } i=x,\ y \tag{2-14}$$

where $X_x(k)=[x_1(k)\quad x_2(k)\quad x_3(k)\quad x_4(k)]^T$, $X_y(k)=[x_5(k)\quad x_6(k)\quad x_7(k)\quad x_8(k)]^T$, $G=e^{AT}, H=\int_0^T e^{At}Bdt, B=[0\quad 0\quad 0\quad 1]^T, C=[0\quad 0\quad 0\quad 1], a=-\dfrac{mg}{m+\frac{J_b}{r_b^2}}$.

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{mg}{m+\frac{J_b}{r_b^2}} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, G=\begin{bmatrix} 1 & T & \dfrac{aT^2}{2} & \dfrac{aT^3}{6} \\ 0 & 1 & aT & \dfrac{aT^2}{2} \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, H=\begin{bmatrix} \dfrac{aT^4}{24} \\ \dfrac{aT^3}{6} \\ \dfrac{T^2}{2} \\ T \end{bmatrix}$$

3. Controller design

For the system (2-14), define the controllable coefficient ξ , so that ξ_i is the smallest, then $\xi=4$.

$$\det([H_i\quad G_iH_i\quad \dots\quad G_i^{\xi_i-1}H_i])\neq 0 \tag{3-1}$$

Define

$$\Lambda=[C\quad CA\quad CA^2\quad CA^3]^T \tag{3-2}$$

$$U_i(k\xi)=[u_i(k\xi)\quad u_i(k\xi+1)\quad u_i(k\xi+2)\quad u_i(k\xi+3)]^T \tag{3-3}$$

$$K=\operatorname{diag}(k_1,\ k_2,\ k_3,\ k_4) \tag{3-4}$$

The sampling time interval ξ is designed as a module in turn, and a modularized control method of active prediction is proposed. In order to get an fully-actuated system, iterate the system (2-14) forward by ξ sampling periods to get

$$X_i((k+1)\xi)=G^{\xi}X_i(k\xi)+\tilde{H}U_i(k\xi) \tag{3-5}$$

where $\tilde{H}=[G^3H\quad G^2H\quad GH\quad H]$.

We can design the control protocol as follows, calculate the control protocol in a module at the sampling time, and use the corresponding control law at the corresponding time, thereby reducing the computational complexity and the computational burden.

$$U_i(k\xi)=\tilde{H}^{-1}(K-G^{\xi})X_i(k\xi)+\Lambda(R_i((k+1)\xi)-KR_i(k\xi)) \tag{3-6}$$

where $R_i(t)=[r_i(t)\quad \dot{r}_i(t)\quad \ddot{r}_i(t)\quad \dddot{r}_i(t)]^T$, $r_i(t)$ is reference trajectory. $k_1,\ k_2,\ k_3,\ k_4\in(0,1)$ are adjustable parameter.

Define

$$\Xi(k)=[a_1\quad a_2\quad a_3\quad a_4],\ a_{\alpha}=\begin{cases} 1, & \alpha=k \\ 0, & otherwise \end{cases} \tag{3-7}$$

Q can be expressed as

$$u_i(k)=\Xi(k)U_i(k) \tag{3-8}$$

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Define the error as

$$e_i(t) = X_i(t) - R_i(t) \tag{3-9}$$

According to (3-5), (3-6), (3-9), it can be calculated that

$$\begin{aligned} e_i((k+1)\xi) &= G^{\xi} X_i(k\xi) + \tilde{H}U_i(k\xi) - R_i((k+1)\xi) \\ &= Ke_i(k\xi) \end{aligned} \tag{3-10}$$

then

$$\| e_i((k+n)\xi) \| = \| K \|^n \| e_i(k\xi) \| \tag{3-11}$$

Because of $\| K \| \in (0,1)$,

$$\lim_{k\to\infty} \|e_i(k)\| = 0 \quad \text{i.e.} \quad \lim_{t\to\infty} \|e_i(t)\| < \delta \tag{3-12}$$

the protocol error converges to the zero domain.

4. Experiment

In order to verify the correctness of the protocol (3-6), simulation verification was carried out, and the experimental tool was MATLAB-Simulink. The parameters are selected as follows: $m=0.263$, $g=9.8$, $r_b=0.02$, $J_b=4.2\times10^{-5}$, $k_1=k_2=k_3=k_4=0.5$, $T=0.05$. The state-space equation can be expressed as:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -7 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ G=\begin{bmatrix} 1 & T & \frac{-7T^2}{2} & \frac{-7T^3}{6} \\ 0 & 1 & -7T & \frac{-7T^2}{2} \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The initial state is selected as the origin, and the reference trajectory is given to a circle with a radius of 10. The simulation results are as follows:

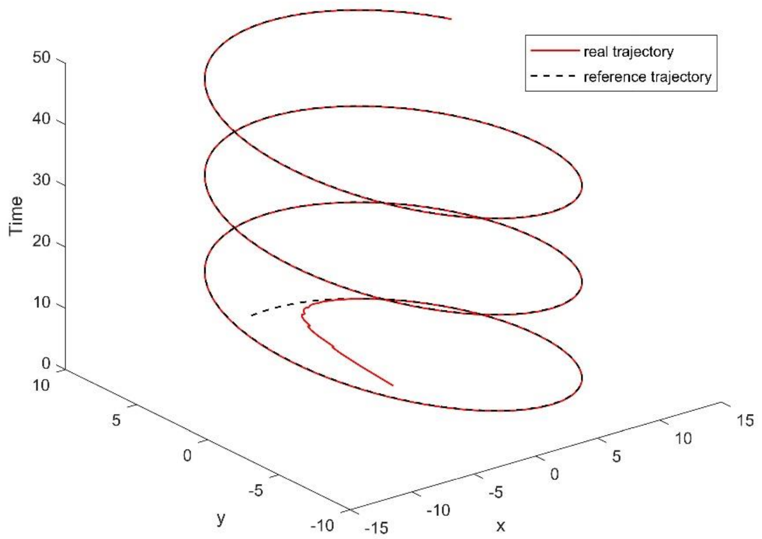

Figure 3: Real trajectory and reference trajectory in simulation

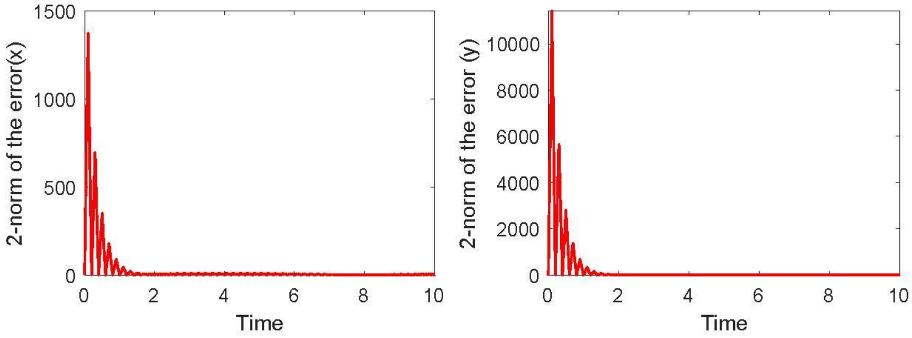

Figure 4: 2-norm of protocol error

It can be seen from Figure 3 that under the control protocol (3-6), the ball can track the reference trajectory well and complete the trajectory tracking. Figure 4 shows the 2-norm of the protocol error, and the 2-norm of the protocol error converges to zero after the simulation time is about 2s. The effectiveness of the proposed protocol is verified by experiments.

5. Conclusion

A modularized control method is proposed for the discrete-time trajectory tracking problem of the ball and plate system. The system is transformed into an all-drive system, and the sampling time is divided into different modules in turn by the controllable coefficient. The control law of the entire module is calculated at the first sampling time of each module, and the rest of the sampling time does not need to be calculated again, thereby reducing the calculation pressure. In the following research, this method will be extended to more general cases, such as formation control.

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