

# Analysis of Vibration Isolation Model Based on Negative Stiffness Principle

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## Abstract

Precision machinery system, the vibration is the important factors influencing the performance of the system, the traditional passive vibration isolation system cannot solve the ultra precision machining and measurement in the fields of the ultra-low frequency vibration problem of broad band, new ultra-low frequency vibration isolator to become the focus of research in various fields, and negative stiffness structure design is the key to the low frequency vibration isolation. Based on the excellent nonlinear characteristics of permanent magnets, a horizontal negative stiffness mechanism composed of two pairs of mutually repulsive permanent magnets is proposed in this paper. Through the analysis of its mechanical characteristics and stiffness characteristics, it is concluded that the negative stiffness mechanism is nonlinear, the system is unstable, and a single negative stiffness structure can not achieve the purpose of vibration isolation. Therefore, the design of passive vibration isolator based on the parallel principle of positive and negative stiffness is the key to realize low frequency vibration isolation.

## Keywords

Vibration isolation; low frequency ;negative stiffness; permanent magnet.

## 1. Introduction

The poor micro vibration will reduce the system performance of precision machinery, so it is very important to realize low frequency and ultra-low frequency vibration isolation. Active control isolation system can effectively solve the problem of micro-vibration, but it has the disadvantages of complex structure and high manufacturing cost. The traditional passive vibration isolation is to reduce the natural frequency of the system to achieve low frequency and ultra-low frequency vibration isolation, usually need to reduce the stiffness of the system, which will bring the problem of excessive deformation of elastic elements, stability and bearing capacity is also affected.

By using the parallel principle of positive and negative stiffness, the negative stiffness mechanism and positive stiffness spring can be connected in parallel, which can effectively reduce the stiffness of the system, meet the requirements of bearing capacity, and improve the vibration isolation ability of the system. The key to the low frequency vibration isolation of the system lies in the realization of the negative stiffness mechanism. Based on this, the negative stiffness vibration isolation device with nonlinear characteristics has a broad application prospect. Experts and scholars at home and abroad have studied various types of negative stiffness mechanisms <sup>[1-11]</sup>: spring type, magnetic type, geometric nonlinear structure type, composite structure type, etc.

After summarizing the existing results, it is found that the non-contact magnetic mechanism can effectively avoid the friction and the lack of compact structure of the mechanism structure,

and the construction is relatively simple in the actual project, and there is no need to redesign the parameters when the external load changes. In this paper, a magneto negative stiffness mechanism based on magnetic repulsion is proposed, and its force characteristics and stiffness characteristics are analyzed.

## 2. Structure Principle of Negative Stiffness Mechanism

### 2.1. Calculation Principle of Magnetic Force

The repulsive force and stiffness between a pair of permanent magnets were calculated according to the theoretical calculation formula derived from the literature<sup>[9]</sup>.

The magnetic force between two repulsive permanent magnets:

$$F = 312500 \cdot B_r^2 \cdot D^2 \cdot \eta^2 \cdot \left( 1 - \frac{L_g}{\sqrt{L_g^2 + D^2}} \right)^2 \tag{1}$$

Where, F is the magnetic force (N);

D is the diameter of permanent magnet (m);

B<sub>r</sub> is the permanent magnet remanence (T);

H is the experimental correction coefficient;

L<sub>g</sub> is the length of air gap (m).

The empirical correction coefficient η determined by the experiment is shown in Table.1:

Table.1 Comparison table of empirical correction coefficient η and L<sub>m</sub> /D

L <sub>m</sub> /D	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
η	0.16	0.28	0.38	0.45	0.51	0.57	0.63	0.66	0.66	0.7

### 2.2. Analysis of Influencing Factors of Magnetic Force

According to the magnetic force formula, the size of the magnetic force is not only related to the nature of the magnet itself (the B<sub>r</sub> value determined by the magnet grade), but also related to the size of the magnet (diameter, thickness) and the length of the air gap between the magnets. In the formula, the parameters include D, B<sub>r</sub>, η, and L<sub>g</sub>, where L<sub>g</sub> is the independent variable and B<sub>r</sub> is a constant.

Select N35 NdFeb, B<sub>r</sub>=1.2t, η=0.51, take a pair of repulsive magnet model as an example, analyze the influence of different parameters on magnetic force:

(1)The influence of different D on magnetic force

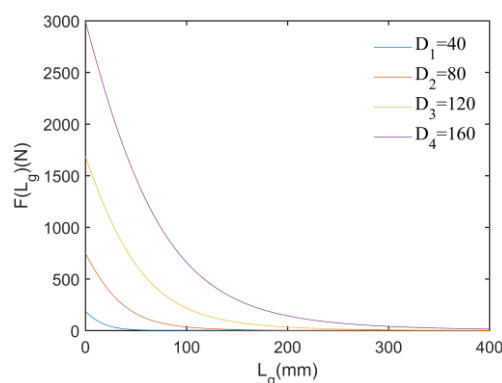


Fig.1 Magnetic force curve with diameter change(η=0.51)

As can be seen from the figure above, the value of the diameter of the magnet has a significant effect on the magnetic force. The larger the diameter, the greater the magnetic force. Under the condition of other magnetic parameters, the smaller the diameter of the magnet, the more obvious the horizontal effect of the intersection.

(2) The influence of empirical correction coefficient on magnetic force

Since the empirical correction coefficient is related to magnet size (diameter and thickness), exploring the effect of empirical coefficient is actually an analysis of the influence of magnet thickness on magnetic force. The smaller the empirical correction coefficient is, the flatter the magnet is.

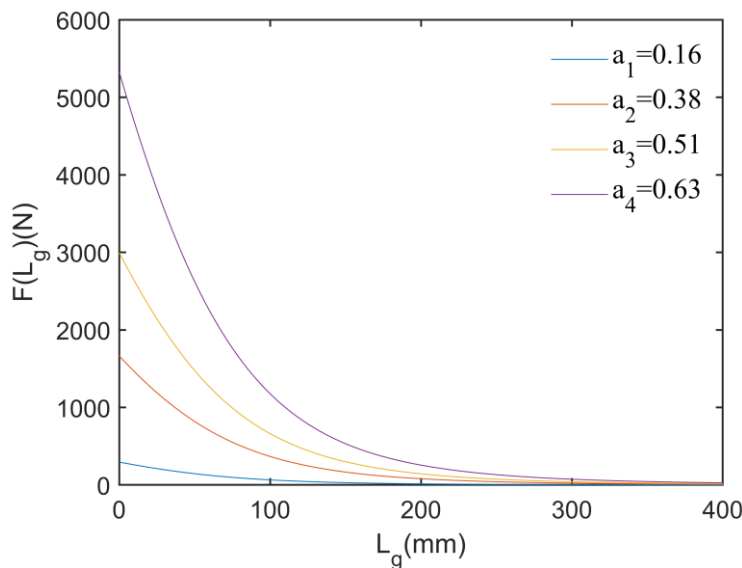


Fig. 2 Force characteristic curve of empirical correction coefficient of change(D=160mm)

As can be seen from the figure above, as the value of the empirical correction coefficient increases, the thickness of the magnet increases, and the magnetic force also increases, but the length of the horizontal section of the middle section decreases, and the stroke range satisfying the low frequency becomes smaller. In the design and calculation, the value of correction coefficient can be reduced under the condition of ensuring the bearing capacity, and the flat design of magnet shape can be sought.

**2.3. Principle of Negative Stiffness Mechanism**

Fig. 3 shows the magneto-negative stiffness structure of the connecting rod. Let the length of the two connecting rods be A, the connecting rod be a rigid massless bar, and the distance between the midpoint O and the external magnet at the balance position be L. When the mass block M is disturbed by an external disturbance, point O moves downward from the initial position, and the distance from the equilibrium position is denoted as y. The Angle between the connecting rod and the central axis of the horizontal magnetic spring is denoted as  $\alpha$ . The two groups of magnetic repulsive springs are connected with the mass block to make a telescopic movement in the horizontal direction. The initial air gap is compressed from  $L_0$  to  $L_x$ , and the mass and friction of the slider are ignored.

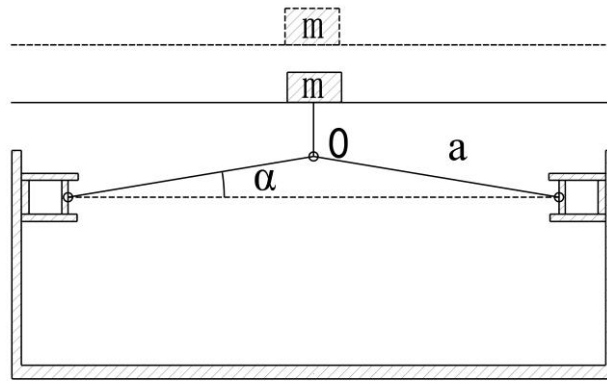


Fig. 3 Schematic diagram of negative stiffness mechanism

At the distance between the mass block and the balance position, the Angle  $\alpha$  between the connecting rod and the negative stiffness magnetic spring is, which can be defined as:

$$\tan \alpha = \frac{y}{L-L_x} = \frac{y}{\sqrt{a^2-y^2}} \tag{2}$$

For dimensionless purposes, we assume that  $L_x$  is the length of the air gap.

After the system is disturbed, point O produces a vertical displacement  $y$ , two pairs of magnetic repulsive springs are symmetrically installed, the resultant force of the horizontal component is zero, and the vertical output force of the negative stiffness mechanism on the mass block  $m$  in the vertical direction is:

$$F_h = 2F_{\bar{F}}y = 2F_{\bar{F}}\tan\alpha \tag{3}$$

According to Equation (3) above, when  $\tan\alpha = 0$ , that is, the vertical displacement  $y = 0$ , the magnetic force direction at this time is horizontal, and there is no component force in the vertical direction, that is,  $F_h = 0$ .

Equation (3) is sorted out and expressed as system parameters:

$$F_{\bar{F}} = A \cdot \left( 1 - \frac{L_g}{\sqrt{L_g^2 + D^2}} \right)^2 \tag{4}$$

write the formula as  $A = 312500 \cdot D^2 \cdot B_r^2 \cdot \eta^2$

$$F_h = 2 \cdot A \left( 1 - \frac{L_x}{\sqrt{L_x^2 + D^2}} \right)^2 \cdot \frac{y}{\sqrt{a^2 - y^2}} \tag{5}$$

Equation (5) is dimensionless, and the following parameters are defined:  $\hat{F}_h = \frac{F_h}{2KL_x}$ ,  $\beta_1 = \frac{\eta^2 B_r^2}{KL_x^{-1}}$ ,  $\beta_2 = \frac{D}{L_x}$ ,  $\hat{y} = \frac{y}{L_x}$ ,  $\beta_3 = \frac{a}{L_x}$ , The output force of the dimensionless negative stiffness mechanism can be obtained as follows:

$$\hat{F}_h = 312500 \cdot \beta_1 \cdot \beta_2^2 \left( 1 - \frac{1}{\sqrt{1 + \beta_2^2}} \right)^2 \cdot \frac{\hat{y}}{\sqrt{\beta_3^2 - \hat{y}^2}} \tag{6}$$

Since  $K \cdot L_x$  is a constant of unit N, it satisfies the principle of dimensionless operation. Vertical stiffness can be obtained from Equation (6) :

$$K_h(y) = \frac{dF_h(y)}{dy} = 2A \cdot \left(1 - \frac{L_X}{\sqrt{L_X^2 + D^2}}\right)^2 \cdot \left(\frac{1}{\sqrt{a^2 - y^2}} + \frac{y^2}{(a^2 - y^2)^{\frac{3}{2}}}\right) \tag{7}$$

According to Equation (7), the vertical stiffness of dimensionless negative stiffness can be written as:

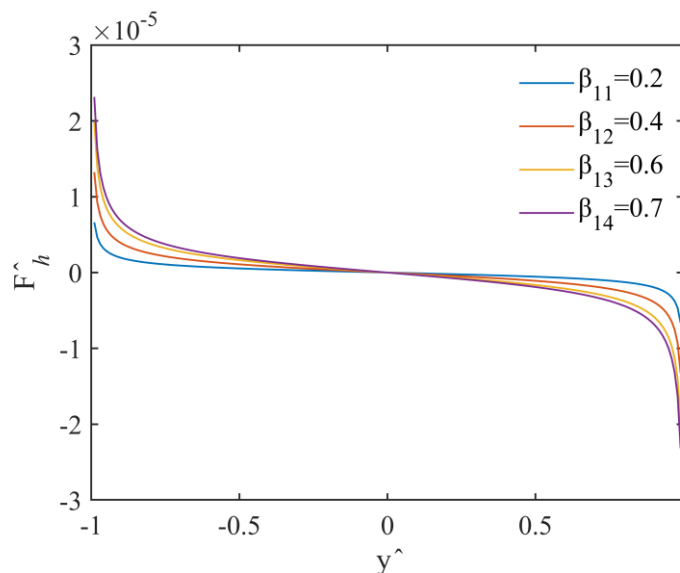
$$\hat{K}_h = 312500 \cdot \beta_1 \cdot \beta_2^2 \left(1 - \frac{1}{\sqrt{1 + \beta_2^2}}\right)^2 \cdot \left(\frac{1}{\sqrt{\beta_3^2 - \hat{y}^2}} + \frac{\hat{y}^2}{(\beta_3^2 - \hat{y}^2)^{\frac{3}{2}}}\right) \tag{8}$$

### 3. Characteristics Analysis of Negative Stiffness Vibration Isolation Mechanism

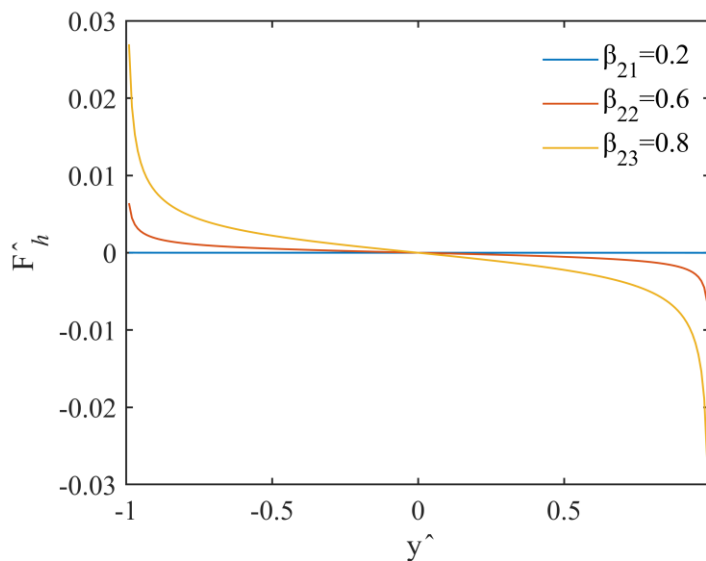
#### 3.1. Analysis of Dimensionless Force Characteristic Curve

According to Equation (6), the curve of dimensionless force and displacement can be obtained, as shown in Fig. 4. It can be seen that the horizontal magnetic spring has nonlinear characteristics of negative stiffness. When point O moves to the equilibrium position, the dimensionless force of the horizontal magnetic spring in the vertical direction is zero. At this time, any slight disturbance will make the system deviate from the equilibrium position, which is an unstable state.

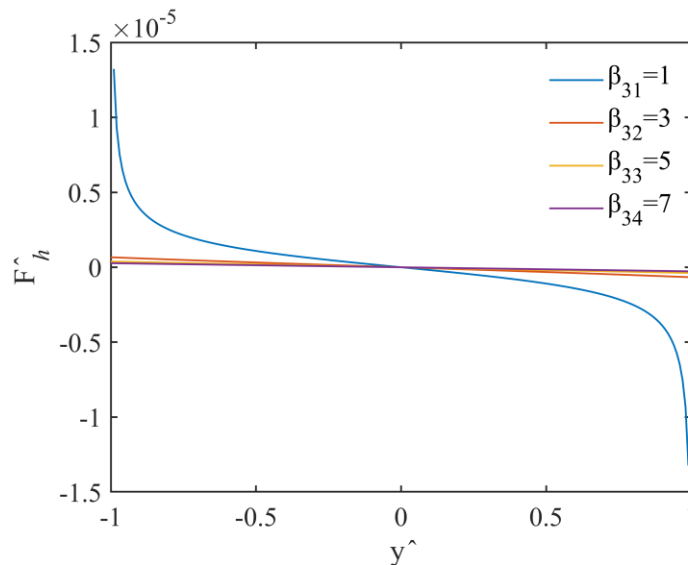
Fig. 4 shows that the parameters  $\beta_1$ ,  $\beta_2$  and  $\beta_3$  all affect the nonlinearity of the structure. When the micro-seismic occurs at the equilibrium position of the negative stiffness mechanism, the dimensionless displacement is small and its linearity is relatively strong. With the increase of the dimensionless displacement, the dimensionless nonlinear becomes stronger. Compared with  $\beta_2$  (diameter of magnet),  $\beta_1$  (experimental correction coefficient) and  $\beta_3$  (bar length A of negative stiffness mechanism) have weaker influence on the value of dimensionless force.



a) Dimensionless force characteristic curve of  $\beta_1$  change ( $\beta_2=0.2$ ,  $\beta_3=1$ )



b) Dimensionless force characteristic curve of  $\beta_2$  variation ( $\beta_1=0.4, \beta_3=1$ )

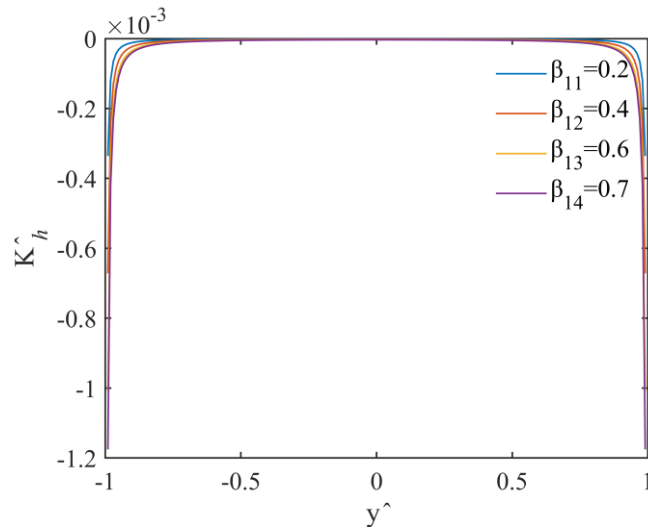


c) Dimensionless force characteristic curve of  $\beta_3$  change ( $\beta_1=0.4, \beta_3=0.2$ )

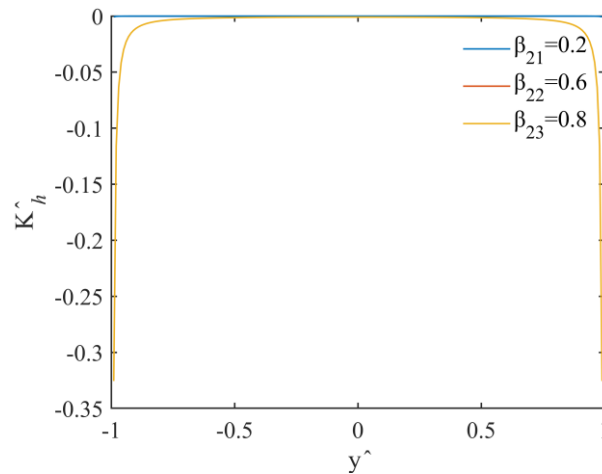
Fig.4 Dimensionless force characteristic curve of negative stiffness mechanism

### 3.2. Analysis of dimensionless stiffness characteristic curve

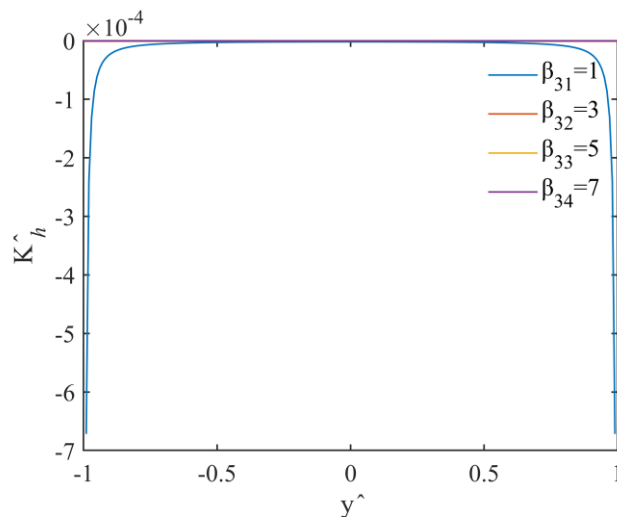
According to Equation (8), curves of dimensionless stiffness and displacement under different parameters can be obtained, as shown in Fig. 4. It can be seen that when the negative stiffness mechanism vibrates slightly near the equilibrium position, the dimensionless stiffness is approximately linear, and the strong non-linearity will be reflected when the negative stiffness mechanism exceeds a certain range.



a) Dimensionless stiffness characteristic curve of  $\beta_1$  change ( $\beta_2=0.2, \beta_3=1$ )



b) Dimensionless stiffness characteristic curve of  $\beta_2$  change ( $\beta_1=0.4, \beta_3=1$ )



c) Dimensionless stiffness characteristic curve of  $\beta_3$  change ( $\beta_1=0.4, \beta_2=0.2$ )

Fig. 5 Dimensionless stiffness characteristic curve of negative stiffness mechanism

Fig.5a shows the influence of parameter  $\beta_1$  (experimental correction coefficient) on the stiffness characteristic curve of the negative stiffness mechanism. It can be seen from the figure that  $\beta_1$  has little influence on the width of the low stiffness region, and the greater the value of  $\beta_1$  is, the greater the stiffness value is. Compared with the three figures, the parameters  $\beta_2$  and

$\beta_3$  affect the width of the low stiffness region of the negative stiffness mechanism. The larger the parameter value, the larger the width of the low stiffness region.

#### 4. Summary

By analyzing the force characteristics, stiffness characteristics and the influence of parameters of the proposed magneto-negative stiffness vibration isolation mechanism composed of permanent magnets placed horizontally, the results show that:

(1) Through the analysis of the factors affecting the magnetic force between two repulsive magnets, it is found that the magnetic force increases with the increase of the magnet diameter and the value of the empirical coefficient.

(2) In the force characteristic curve of the negative stiffness mechanism, the larger the values of  $\beta_1$  and  $\beta_2$ , the greater the magnetic force of the system; The smaller the value of  $\beta_3$ , the greater the magnetic force. In the stiffness characteristic curve, the parameters  $\beta_2$  and  $\beta_3$  affect the width of the low stiffness region of the negative stiffness mechanism. The larger the value is, the larger the width of the low stiffness region will be.

(3) The characteristic curve of the magneto-negative stiffness structure is nonlinear and the system is unstable. The purpose of vibration isolation cannot be realized only by relying on the negative stiffness structure. Therefore, it is necessary to connect the negative stiffness mechanism with the positive stiffness element in parallel, so as to effectively isolate vibration, ensure the bearing capacity of the system and improve the stability of the system.

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